Chapter 6
Valuing Bonds
Chapter Outline

6.1 Bond Cash Flows, Prices, and Yields
6.2 Dynamic Behavior of Bond Prices
6.3 The Yield Curve and Bond Arbitrage
6.4 Corporate Bonds
6.5 Sovereign Bonds
6.1 Bond Cash Flows, Prices, and Yields

• Bond Terminology
  – Bond Certificate
    • States the terms of the bond
  – Maturity Date
    • Final repayment date
  – Term
    • The time remaining until the repayment date
  – Coupon
    • Promised interest payments
• Bond Terminology
  – Face Value
    • Notional amount used to compute the interest payments
  – Coupon Rate
    • Determines the amount of each coupon payment, expressed as an APR
  – Coupon Payment

\[
CPN = \frac{\text{Coupon Rate} \times \text{Face Value}}{\text{Number of Coupon Payments per Year}}
\]
Zero-Coupon Bonds

• Zero-Coupon Bond
  – Does not make coupon payments
  – Always sells at a **discount** (a price lower than face value), so they are also called **pure discount bonds**
  – **Treasury Bills** are U.S. government zero-coupon bonds with a maturity of up to one year.
Zero-Coupon Bonds (cont'd)

• Suppose that a one-year, risk-free, zero-coupon bond with a $100,000 face value has an initial price of $96,618.36. The cash flows would be:

- Although the bond pays no “interest,” your compensation is the difference between the initial price and the face value.
Zero-Coupon Bonds (cont'd)

• Yield to Maturity
  – The discount rate that sets the present value of the promised bond payments equal to the current market price of the bond.
  • Price of a Zero-Coupon bond

\[ P = \frac{FV}{(1 + YTM^n)^n} \]
Zero-Coupon Bonds (cont'd)

- Yield to Maturity
  - For the one-year zero coupon bond:

\[
96,618.36 = \frac{100,000}{(1 + YTM_1)}
\]

\[
1 + YTM_1 = \frac{100,000}{96,618.36} = 1.035
\]

- Thus, the YTM is 3.5%.
Zero-Coupon Bonds (cont'd)

• Yield to Maturity
  – Yield to Maturity of an \( n \)-Year Zero-Coupon Bond

\[
YTM_n = \left( \frac{FV}{P} \right)^{\frac{1}{n}} - 1
\]
**Textbook Example 6.1**

### Yields for Different Maturities

#### Problem
Suppose the following zero-coupon bonds are trading at the prices shown below per $100 face value. Determine the corresponding spot interest rates that determine the zero coupon yield curve.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>1 Year</th>
<th>2 Years</th>
<th>3 Years</th>
<th>4 Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>$96.62</td>
<td>$92.45</td>
<td>$87.63</td>
<td>$83.06</td>
</tr>
</tbody>
</table>
Textbook Example 6.1 (cont'd)

Solution
Using Eq. 6.3, we have

\[ r_1 = YTM_1 = \frac{100}{96.62} - 1 = 3.50\% \]

\[ r_2 = YTM_2 = (\frac{100}{92.45})^{1/2} - 1 = 4.00\% \]

\[ r_3 = YTM_3 = (\frac{100}{87.63})^{1/3} - 1 = 4.50\% \]

\[ r_4 = YTM_4 = (\frac{100}{83.06})^{1/4} - 1 = 4.75\% \]
Alternative Example 6.1

• Problem
  – Suppose that the following zero-coupon bonds are selling at the prices shown below per $100 face value. Determine the corresponding yield to maturity for each bond.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>1 year</th>
<th>2 years</th>
<th>3 years</th>
<th>4 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price</td>
<td>$98.04</td>
<td>$95.18</td>
<td>$91.51</td>
<td>$87.14</td>
</tr>
</tbody>
</table>
• Solution:

\[
\text{YTM} = \frac{100}{98.04} - 1 = 0.02 = 2\%
\]
\[
\text{YTM} = \frac{100}{95.18}^{\frac{1}{2}} - 1 = 0.025 = 2.5\%
\]
\[
\text{YTM} = \frac{100}{91.51}^{\frac{1}{3}} - 1 = 0.03 = 3\%
\]
\[
\text{YTM} = \frac{100}{87.14}^{\frac{1}{4}} - 1 = 0.035 = 3.5\%
\]
Zero-Coupon Bonds (cont'd)

• Risk-Free Interest Rates

  – A default-free zero-coupon bond that matures on date $n$ provides a risk-free return over the same period. Thus, the Law of One Price guarantees that the risk-free interest rate equals the yield to maturity on such a bond.

  – Risk-Free Interest Rate with Maturity $n$

\[ r_n = YTM_n \]
Zero-Coupon Bonds (cont'd)

• Risk-Free Interest Rates
  – Spot Interest Rate
    • Another term for a default-free, zero-coupon yield
  – Zero-Coupon Yield Curve
    • A plot of the yield of risk-free zero-coupon bonds as a function of the bond’s maturity date
Coupon Bonds

- Coupon Bonds
  - Pay face value at maturity
  - Pay regular coupon interest payments

- Treasury Notes
  - U.S. Treasury coupon security with original maturities of 1–10 years

- Treasury Bonds
  - U.S. Treasury coupon security with original maturities over 10 years
Textbook Example 6.2

The Cash Flows of a Coupon Bond

Problem
The U.S. Treasury has just issued a five-year, $1000 bond with a 5% coupon rate and semiannual coupons. What cash flows will you receive if you hold this bond until maturity?
Solution
The face value of this bond is $1000. Because this bond pays coupons semiannually, from Eq. 6.1, you will receive a coupon payment every six months of \( CPN = 1000 \times \frac{5\%}{2} = 25 \). Here is the timeline, based on a six-month period:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>...</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$25</td>
<td>$25</td>
<td>$25</td>
<td>$25</td>
<td></td>
<td>$25 + $1000</td>
</tr>
</tbody>
</table>

Note that the last payment occurs five years (10 six-month periods) from now and is composed of both a coupon payment of $25 and the face value payment of $1000.
The U.S. Treasury has just issued a ten-year, $1000 bond with a 4% coupon and semi-annual coupon payments. What cash flows will you receive if you hold the bond until maturity?
Alternative Example 6.2 (cont'd)

The face value of this bond is $1000. Because this bond pays coupons semiannually, from Eq. 8.1 you will receive a coupon payment every six months of \( \text{CPN} = 1000 \times 4\%/2 = 20 \). Here is the timeline, based on a six-month period:

![Timeline diagram showing payments at 0, 1, 2, 3, ..., 20 periods with amounts $20, $20, $20, and $20 + 1000.]

Note that the last payment occurs ten years (twenty six-month periods) from now and is composed of both a coupon payment of $20 and the face value payment of $1000.
Coupon Bonds (cont'd)

• Yield to Maturity
  – The YTM is the *single* discount rate that equates the present value of the bond’s remaining cash flows to its current price.

  $$ P = CPN \times \frac{1}{y} \left(1 - \frac{1}{(1 + y)^N}\right) + \frac{FV}{(1 + y)^N} $$

  \[0\] \[1\] \[2\] \[3\] \[\ldots\] \[N\]
  \[-P\] \[CPN\] \[CPN\] \[CPN\] \[\ldots\] \[CPN + FV\]
Textbook Example 6.3

Computing the Yield to Maturity of a Coupon Bond

Problem
Consider the five-year, $1000 bond with a 5% coupon rate and semiannual coupons described in Example 6.2. If this bond is currently trading for a price of $957.35, what is the bond’s yield to maturity?
Textbook Example 6.3 (cont'd)

Solution
Because the bond has 10 remaining coupon payments, we compute its yield $y$ by solving:

$$957.35 = 25 \times \frac{1}{y} \left( 1 - \frac{1}{(1 + y)^{10}} \right) + \frac{1000}{(1 + y)^{10}}$$

We can solve it by trial-and-error or by using the annuity spreadsheet:

<table>
<thead>
<tr>
<th>NPER</th>
<th>RATE</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
<th>Excel Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given</td>
<td>10</td>
<td>-957.35</td>
<td>25</td>
<td>1,000</td>
<td>=RATE(10,25,-957.35,1000)</td>
</tr>
<tr>
<td>Solve for PV</td>
<td>3.00%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Therefore, $y = 3\%$. Because the bond pays coupons semiannually, this yield is for a six-month period. We convert it to an APR by multiplying by the number of coupon payments per year. Thus the bond has a yield to maturity equal to a 6\% APR with semiannual compounding.
Computing a Bond Price from Its Yield to Maturity

Problem
Consider again the five-year, $1000 bond with a 5% coupon rate and semiannual coupons presented in Example 6.3. Suppose you are told that its yield to maturity has increased to 6.30% (expressed as an APR with semiannual compounding). What price is the bond trading for now?
**Solution**
Given the yield, we can compute the price using Eq. 6.5. First, note that a 6.30% APR is equivalent to a semiannual rate of 3.15%. Therefore, the bond price is

\[
P = 25 \times \frac{1}{0.0315} \left(1 - \frac{1}{1.0315^{10}}\right) + \frac{1000}{1.0315^{10}} = 944.98
\]

We can also use the annuity spreadsheet:

<table>
<thead>
<tr>
<th></th>
<th>NPER</th>
<th>RATE</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
<th>Excel Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given</td>
<td>10</td>
<td>3.15%</td>
<td>25</td>
<td>1,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solve for PV</td>
<td></td>
<td></td>
<td><strong>-944.98</strong></td>
<td></td>
<td></td>
<td>= PV(0.0315, 10, 25, 1,000)</td>
</tr>
</tbody>
</table>
6.2 Dynamic Behavior of Bond Prices

- **Discount**
  - A bond is selling at a **discount** if the price is less than the face value.

- **Par**
  - A bond is selling at **par** if the price is equal to the face value.

- **Premium**
  - A bond is selling at a **premium** if the price is greater than the face value.
Discounts and Premiums

- If a coupon bond trades at a discount, an investor will earn a return both from receiving the coupons and from receiving a face value that exceeds the price paid for the bond.
  
  - If a bond trades at a discount, its yield to maturity will exceed its coupon rate.
Discounts and Premiums (cont'd)

- If a coupon bond trades at a premium it will earn a return from receiving the coupons but this return will be diminished by receiving a face value less than the price paid for the bond.

- Most coupon bonds have a coupon rate so that the bonds will *initially* trade at, or very close to, par.
**Discounts and Premiums (cont'd)**

**Table 6.1** Bond Prices Immediately After a Coupon Payment

<table>
<thead>
<tr>
<th>When the bond price is</th>
<th>We say the bond trades</th>
<th>This occurs when</th>
</tr>
</thead>
<tbody>
<tr>
<td>greater than the face value</td>
<td>“above par” or “at a premium”</td>
<td>Coupon Rate &gt; Yield to Maturity</td>
</tr>
<tr>
<td>equal to the face value</td>
<td>“at par”</td>
<td>Coupon Rate = Yield to Maturity</td>
</tr>
<tr>
<td>less than the face value</td>
<td>“below par” or “at a discount”</td>
<td>Coupon Rate &lt; Yield to Maturity</td>
</tr>
</tbody>
</table>
Determining the Discount or Premium of a Coupon Bond

Problem
Consider three 30-year bonds with annual coupon payments. One bond has a 10% coupon rate, one has a 5% coupon rate, and one has a 3% coupon rate. If the yield to maturity of each bond is 5%, what is the price of each bond per $100 face value? Which bond trades at a premium, which trades at a discount, and which trades at par?
Textbook Example 6.5 (cont'd)

Solution
We can compute the price of each bond using Eq. 6.5. Therefore, the bond prices are

\[ P(10\% \text{ coupon}) = 10 \times \frac{1}{0.05} \left( 1 - \frac{1}{1.05^{30}} \right) + \frac{100}{1.05^{30}} = 176.86 \quad \text{(trades at a premium)} \]

\[ P(5\% \text{ coupon}) = 5 \times \frac{1}{0.05} \left( 1 - \frac{1}{1.05^{30}} \right) + \frac{100}{1.05^{30}} = 100.00 \quad \text{(trades at par)} \]

\[ P(3\% \text{ coupon}) = 3 \times \frac{1}{0.05} \left( 1 - \frac{1}{1.05^{30}} \right) + \frac{100}{1.05^{30}} = 69.26 \quad \text{(trades at a discount)} \]
Time and Bond Prices

• Holding all other things constant, a bond’s yield to maturity will not change over time.

• Holding all other things constant, the price of discount or premium bond will move towards par value over time.

• If a bond’s yield to maturity has not changed, then the IRR of an investment in the bond equals its yield to maturity even if you sell the bond early.
The Effect of Time on the Price of a Coupon Bond

Problem
Consider a 30-year bond with a 10% coupon rate (annual payments) and a $100 face value. What is the initial price of this bond if it has a 5% yield to maturity? If the yield to maturity is unchanged, what will the price be immediately before and after the first coupon is paid?
Solution
We computed the price of this bond with 30 years to maturity in Example 6.5:

\[ P = 10 \times \frac{1}{0.05} \left( 1 - \frac{1}{1.05^{30}} \right) + \frac{100}{1.05^{30}} = \$176.86 \]

Now consider the cash flows of this bond in one year, immediately before the first coupon is paid. The bond now has 29 years until it matures, and the timeline is as follows:

\[
\begin{array}{cccccc}
0 & & 1 & & 2 & \cdots \ && 29 \\
$10 & & $10 & & $10 & \cdots \ && $10 + $100
\end{array}
\]

Again, we compute the price by discounting the cash flows by the yield to maturity. Note that there is a cash flow of $10 at date zero, the coupon that is about to be paid. In this case, it is easiest to treat the first coupon separately and value the remaining cash flows as in Eq. 6.5:

\[ P \text{(just before first coupon)} = 10 + 10 \times \frac{1}{0.05} \left( 1 - \frac{1}{1.05^{29}} \right) + \frac{100}{1.05^{29}} = \$185.71 \]
Note that the bond price is higher than it was initially. It will make the same total number of coupon payments, but an investor does not need to wait as long to receive the first one. We could also compute the price by noting that because the yield to maturity remains at 5% for the bond, investors in the bond should earn a return of 5% over the year: $176.86 \times 1.05 = 185.71$.

What happens to the price of the bond just after the first coupon is paid? The timeline is the same as that given earlier, except the new owner of the bond will not receive the coupon at date zero. Thus, just after the coupon is paid, the price of the bond (given the same yield to maturity) will be

$$P(\text{just after first coupon}) = 10 \times \frac{1}{0.05} \left(1 - \frac{1}{1.05^{29}}\right) + \frac{100}{1.05^{29}} = 175.71$$

The price of the bond will drop by the amount of the coupon ($10$) immediately after the coupon is paid, reflecting the fact that the owner will no longer receive the coupon. In this case, the price is lower than the initial price of the bond. Because there are fewer coupon payments remaining, the premium investors will pay for the bond declines. Still, an investor who buys the bond initially, receives the first coupon, and then sells it earns a 5% return if the bond’s yield does not change: $(10 + 175.71)/176.86 = 1.05$. 

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Figure 6.1  The Effect of Time on Bond Prices
Accrued Interest (German)

Accrued Interest

\[
\text{Accrued Int.} = \text{Coupon Amount} \times \left( \frac{\text{days since last coupon payment}}{\text{days in current coupon period}} \right)
\]

\[
\text{Stückzinsen} = \text{Kuponbetrug} \times \left( \frac{\text{Tage seit letzter Kuponzahlung}}{\text{Tage in aktueller Kuponperiode}} \right)
\]

Clean Price = Dirty Price - Accrued Interest
Interest Rate Changes and Bond Prices

• There is an inverse relationship between interest rates and bond prices.
  – As interest rates and bond yields rise, bond prices fall.
  – As interest rates and bond yields fall, bond prices rise.
Interest Rate Changes and Bond Prices (cont'd)

• The sensitivity of a bond’s price to changes in interest rates is measured by the bond’s duration.
  – Bonds with high durations are highly sensitive to interest rate changes.
  – Bonds with low durations are less sensitive to interest rate changes.
Textbook Example 6.7

The Interest Rate Sensitivity of Bonds

Problem
Consider a 15-year zero-coupon bond and a 30-year coupon bond with 10% annual coupons. By what percentage will the price of each bond change if its yield to maturity increases from 5% to 6%?
Textbook Example 6.7 (cont'd)

**Solution**

First, we compute the price of each bond for each yield to maturity:

<table>
<thead>
<tr>
<th>Yield to Maturity</th>
<th>15-Year, Zero-Coupon Bond</th>
<th>30-Year, 10% Annual Coupon Bond</th>
</tr>
</thead>
<tbody>
<tr>
<td>5%</td>
<td>( \frac{100}{1.05^{15}} = $48.10 )</td>
<td>( 10 \times \frac{1}{0.05} \left( 1 - \frac{1}{1.05^{30}} \right) + \frac{100}{1.05^{30}} = $176.86 )</td>
</tr>
<tr>
<td>6%</td>
<td>( \frac{100}{1.06^{15}} = $41.73 )</td>
<td>( 10 \times \frac{1}{0.06} \left( 1 - \frac{1}{1.06^{30}} \right) + \frac{100}{1.06^{30}} = $155.06 )</td>
</tr>
</tbody>
</table>

The price of the 15-year zero-coupon bond changes by \((41.73 - 48.10)/48.10 = -13.2\%\) if its yield to maturity increases from 5% to 6%. For the 30-year bond with 10% annual coupons, the price change is \((155.06 - 176.86)/176.86 = -12.3\%\). Even though the 30-year bond has a longer maturity, because of its high coupon rate, its sensitivity to a change in yield is actually less than that of the 15-year zero coupon bond.
Figure 6.2  Yield to Maturity and Bond Price Fluctuations Over Time
6.3 The Yield Curve and Bond Arbitrage

- Using the Law of One Price and the yields of default-free zero-coupon bonds, one can determine the price and yield of any other default-free bond.

- The yield curve provides sufficient information to evaluate all such bonds.
Replicating a three-year $1000 bond that pays 10% annual coupon using three zero-coupon bonds:

<table>
<thead>
<tr>
<th>Time</th>
<th>Coupon bond:</th>
<th>1-year zero:</th>
<th>2-year zero:</th>
<th>3-year zero:</th>
<th>Zero-coupon Bond portfolio:</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$100</td>
<td>$100</td>
<td>$100</td>
<td>$1100</td>
<td>$100</td>
</tr>
<tr>
<td>1</td>
<td>$100</td>
<td>$100</td>
<td>$100</td>
<td>$1100</td>
<td>$100</td>
</tr>
<tr>
<td>2</td>
<td>$100</td>
<td>$100</td>
<td>$100</td>
<td>$1100</td>
<td>$100</td>
</tr>
<tr>
<td>3</td>
<td>$1100</td>
<td>$100</td>
<td>$100</td>
<td>$1100</td>
<td>$1100</td>
</tr>
</tbody>
</table>

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Replicating a Coupon Bond (cont'd)

• Yields and Prices (per $100 Face Value) for Zero Coupon Bonds

Table 6.2  Yields and Prices (per $100 Face Value) for Zero-Coupon Bonds

<table>
<thead>
<tr>
<th>Maturity</th>
<th>1 year</th>
<th>2 years</th>
<th>3 years</th>
<th>4 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>YTM</td>
<td>3.50%</td>
<td>4.00%</td>
<td>4.50%</td>
<td>4.75%</td>
</tr>
<tr>
<td>Price</td>
<td>$96.62</td>
<td>$92.45</td>
<td>$87.63</td>
<td>$83.06</td>
</tr>
</tbody>
</table>
Replicating a Coupon Bond (cont'd)

<table>
<thead>
<tr>
<th>Zero-Coupon Bond</th>
<th>Face Value Required</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>100</td>
<td>96.62</td>
</tr>
<tr>
<td>2 years</td>
<td>100</td>
<td>92.45</td>
</tr>
<tr>
<td>3 years</td>
<td>1100</td>
<td>$11\times 87.63 = 963.93</td>
</tr>
</tbody>
</table>

**Total Cost:** $1153.00

- By the Law of One Price, the three-year coupon bond must trade for a price of $1153.
Valuing a Coupon Bond
Using Zero-Coupon Yields

• The price of a coupon bond must equal the present value of its coupon payments and face value.
  – Price of a Coupon Bond

\[ PV = PV(\text{Bond Cash Flows}) \]

\[ = \frac{CPN}{1 + YTM_1} + \frac{CPN}{(1 + YTM_2)^2} + \cdots + \frac{CPN + FV}{(1 + YTM_n)^n} \]

\[ P = \frac{100}{1.035} + \frac{100}{1.04^2} + \frac{100 + 1000}{1.045^3} = \$1153 \]
Coupon Bond Yields

- Given the yields for zero-coupon bonds, we can price a coupon bond.

\[ P = 1153 = \frac{100}{(1 + y)} + \frac{100}{(1 + y)^2} + \frac{100 + 1000}{(1 + y)^3} \]

\[ P = \frac{100}{1.0444} + \frac{100}{1.0444^2} + \frac{100 + 1000}{1.0444^3} = $1153 \]

<table>
<thead>
<tr>
<th>NPER</th>
<th>RATE</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
<th>Excel Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given</td>
<td>3</td>
<td>−1,153</td>
<td>100</td>
<td>1,000</td>
<td>= RATE(3,100,−1153,1000)</td>
</tr>
<tr>
<td>Solve for Rate</td>
<td>4.44%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

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Textbook Example 6.8

Yields on Bonds with the Same Maturity

Problem
Given the following zero-coupon yields, compare the yield to maturity for a three-year, zero-coupon bond; a three-year coupon bond with 4% annual coupons; and a three-year coupon bond with 10% annual coupons. All of these bonds are default free.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>1 year</th>
<th>2 years</th>
<th>3 years</th>
<th>4 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zero-coupon YTM</td>
<td>3.50%</td>
<td>4.00%</td>
<td>4.50%</td>
<td>4.75%</td>
</tr>
</tbody>
</table>
Solution

From the information provided, the yield to maturity of the three-year, zero-coupon bond is 4.50%. Also, because the yields match those in Table 6.2, we already calculated the yield to maturity for the 10% coupon bond as 4.44%. To compute the yield for the 4% coupon bond, we first need to calculate its price. Using Eq. 6.6, we have

\[
P = \frac{40}{1.035} + \frac{40}{1.04^2} + \frac{40 + 1000}{1.045^3} = $986.98
\]

The price of the bond with a 4% coupon is $986.98. From Eq. 6.5, its yield to maturity solves the following equation:

\[
986.98 = \frac{40}{1 + y} + \frac{40}{(1 + y)^2} + \frac{40 + 1000}{(1 + y)^3}
\]

We can calculate the yield to maturity using the annuity spreadsheet:

<table>
<thead>
<tr>
<th>NPER</th>
<th>RATE</th>
<th>PV</th>
<th>PMT</th>
<th>FV</th>
<th>Excel Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Given</td>
<td>3</td>
<td>-986.98</td>
<td>40</td>
<td>1,000</td>
<td></td>
</tr>
<tr>
<td>Solve for Rate</td>
<td>4.47%</td>
<td></td>
<td></td>
<td></td>
<td>= RATE(3,40,-986.98,1000)</td>
</tr>
</tbody>
</table>

To summarize, for the three-year bonds considered

<table>
<thead>
<tr>
<th>Coupon rate</th>
<th>0%</th>
<th>4%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>YTM</td>
<td>4.50%</td>
<td>4.47%</td>
<td>4.44%</td>
</tr>
</tbody>
</table>
Treasury Yield Curves

• Treasury Coupon-Paying Yield Curve
  – Often referred to as “the yield curve”

• On-the-Run Bonds
  – Most recently issued bonds
  – The yield curve is often a plot of the yields on these bonds.
6.4 Corporate Bonds

- Corporate Bonds
  - Issued by corporations

- Credit Risk
  - Risk of default
Corporate Bond Yields

• Investors pay less for bonds with credit risk than they would for an otherwise identical default-free bond.

• The yield of bonds with credit risk will be higher than that of otherwise identical default-free bonds.
Corporate Bond Yields (cont'd)

• No Default
  – Consider a 1-year, zero coupon Treasury Bill with a YTM of 4%.
  • What is the price?

\[
P = \frac{1000}{1 + YTM_1} = \frac{1000}{1.04} = \$961.54
\]
Corporate Bond Yields (cont'd)

- Certain Default

  - Suppose now bond issuer will pay 90% of the obligation.
  
    - What is the price?

\[
P = \frac{900}{1 + YTM_1} = \frac{900}{1.04} = 865.38
\]
Corporate Bond Yields (cont'd)

• Certain Default
  – When computing the yield to maturity for a bond with certain default, the promised rather than the actual cash flows are used.

\[
YTM = \frac{FV}{P} - 1 = \frac{1000}{865.38} - 1 = 15.56%
\]

\[
\frac{900}{865.38} = 1.04
\]
Corporate Bond Yields (cont'd)

- Certain Default
  
  The yield to maturity of a certain default bond is not equal to the expected return of investing in the bond. The yield to maturity will always be higher than the expected return of investing in the bond.
Corporate Bond Yields (cont'd)

• Risk of Default
  – Consider a one-year, $1000, zero-coupon bond issued. Assume that the bond payoffs are uncertain.
    • There is a 50% chance that the bond will repay its face value in full and a 50% chance that the bond will default and you will receive $900. Thus, you would expect to receive $950.
    • Because of the uncertainty, the discount rate is 5.1%.
Corporate Bond Yields (cont'd)

• Risk of Default
  – The price of the bond will be
    \[ P = \frac{950}{1.051} = \$903.90 \]
  – The yield to maturity will be
    \[ YTM = \frac{FV}{P} - 1 = \frac{1000}{903.90} - 1 = .1063 \]
Corporate Bond Yields (cont'd)

- Risk of Default
  - A bond’s expected return will be less than the yield to maturity if there is a risk of default.
  - A higher yield to maturity does not necessarily imply that a bond’s expected return is higher.
Corporate Bond Yields (cont'd)

Table 6.3  Price, Expected Return, and Yield to Maturity of a One-Year, Zero-Coupon Avant Bond with Different Likelihoods of Default

<table>
<thead>
<tr>
<th>Avant Bond (1-year, zero-coupon)</th>
<th>Bond Price</th>
<th>Yield to Maturity</th>
<th>Expected Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Default Free</td>
<td>$961.54</td>
<td>4.00%</td>
<td>4%</td>
</tr>
<tr>
<td>50% Chance of Default</td>
<td>$903.90</td>
<td>10.63%</td>
<td>5.1%</td>
</tr>
<tr>
<td>Certain Default</td>
<td>$865.38</td>
<td>15.56%</td>
<td>4%</td>
</tr>
</tbody>
</table>
Bond Ratings

• Investment Grade Bonds

• Speculative Bonds
  – Also known as Junk Bonds or High-Yield Bonds
**Table 6.4  Bond Ratings**

<table>
<thead>
<tr>
<th>Rating*</th>
<th>Description (Moody’s)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Investment Grade Debt</strong></td>
<td></td>
</tr>
<tr>
<td>Aaa/AAA</td>
<td>Judged to be of the best quality. They carry the smallest degree of investment risk and are generally referred to as “gilt edged.” Interest payments are protected by a large or an exceptionally stable margin and principal is secure. While the various protective elements are likely to change, such changes as can be visualized are most unlikely to impair the fundamentally strong position of such issues.</td>
</tr>
<tr>
<td>Aa/AA</td>
<td>Judged to be of high quality by all standards. Together with the Aaa group, they constitute what are generally known as high-grade bonds. They are rated lower than the best bonds because margins of protection may not be as large as in Aaa securities or fluctuation of protective elements may be of greater amplitude or there may be other elements present that make the long-term risk appear somewhat larger than the Aaa securities.</td>
</tr>
<tr>
<td>A/A</td>
<td>Possess many favorable investment attributes and are considered as upper-medium-grade obligations. Factors giving security to principal and interest are considered adequate, but elements may be present that suggest a susceptibility to impairment some time in the future.</td>
</tr>
<tr>
<td>Baa/BBB</td>
<td>Are considered as medium-grade obligations (i.e., they are neither highly protected nor poorly secured). Interest payments and principal security appear adequate for the present but certain protective elements may be lacking or may be characteristically unreliable over any great length of time. Such bonds lack outstanding investment characteristics and, in fact, have speculative characteristics as well.</td>
</tr>
</tbody>
</table>
### Table 6.4  Bond Ratings (cont’d)

<table>
<thead>
<tr>
<th>Speculative Bonds</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ba/BB</td>
<td>Judged to have speculative elements; their future cannot be considered as well assured. Often the protection of interest and principal payments may be very moderate, and thereby not well safeguarded during both good and bad times over the future. Uncertainty of position characterizes bonds in this class.</td>
</tr>
<tr>
<td>B/B</td>
<td>Generally lack characteristics of the desirable investment. Assurance of interest and principal payments of maintenance of other terms of the contract over any long period of time may be small.</td>
</tr>
<tr>
<td>Caa/CCC</td>
<td>Are of poor standing. Such issues may be in default or there may be present elements of danger with respect to principal or interest.</td>
</tr>
<tr>
<td>Ca/CC</td>
<td>Are speculative in a high degree. Such issues are often in default or have other marked shortcomings.</td>
</tr>
<tr>
<td>C/C, D</td>
<td>Lowest-rated class of bonds, and issues so rated can be regarded as having extremely poor prospects of ever attaining any real investment standing.</td>
</tr>
</tbody>
</table>

*Ratings: Moody’s/Standard & Poor’s  
Source: [www.moodys.com](http://www.moodys.com)
Corporate Yield Curves

• Default Spread
  – Also known as Credit Spread
  – The difference between the yield on corporate bonds and Treasury yields
Figure 6.3 Corporate Yield Curves for Various Ratings, June 2012

Source: Bloomberg
Figure 6.4
Yield Spreads and the Financial Crisis

Source: Bloomberg.com
6.5 Sovereign Bonds

- Bonds issued by national governments
  - U.S. Treasury securities are generally considered to be default free
  - All sovereign bonds are not default free
    - e.g. Greece defaulted on its outstanding debt in 2012
  - Importance of inflation expectations
    - Potential to “inflate away” the debt
  - European sovereign debt, the EMU, and the ECB
Figure 6.5 Percent of Countries in Default or Restructuring Debt, 1800–2006

Figure 6.6 European Government Bond Yields, 1963–2011

Source: Federal Reserve Economic Data, research.stlouisfed.org/fred2.
Government Bonds in Germany
Overview

German Government securities can be classified according to maturity and whether they are listed on a stock exchange or not. They offer investment opportunities for virtually any time horizon.